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Mach's Educational Theory and Practice

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Mach's life in Habsburg Austria, his commitment to the Enlightenment tradition, his philosophy of science, his phenomenalism, his psychology, and his historical studies – are all important for understanding his views on education; all are important for what might be called his educational theory and practice. Mach championed a new Austrian school system with the creation of the new *Einheitsschule* where integrated education in the humanities and the sciences could occur; he promoted new methods of teaching based on psychological research on concept acquisition; and he advocated the restructuring of science curricula based on history of the sciences and informed by philosophy of the sciences.

Mach had the characteristic Enlightenment broad-view of science education: it encompassed both formal and informal education; it was subsumed under the category of the 'popularisation of science'. In 1866, during his first university appointment at Graz, he wrote:

> Once a part of science belongs to the [research] literature, a second task remains, which is to popularize it, if possible. This second task also has its importance, but it is a difficult one. It has its importance, because – regardless of the distribution of knowledge that increases its value – it is not unimportant either for the further development of science itself how much knowledge has been disseminated into the public. The difficulty is to know the soil very well in which one wants to plant the knowledge. (In Siemsen 2014, pp.2336-37)

He recognises that the growth of scientific knowledge, of future research, is dependent on the popularisation or spread of extant knowledge. At the time, and even subsequently, this was a rarely acknowledged dependency: scientific research in laboratories depends directly and indirectly on science teaching in classrooms.[1] .

Mach did not write any book titled *Educational Theory and Practice* however his ideas on the social function of schools, on curriculum and on pedagogy can be discerned from four sources: his historical books that in part arose from his own teaching, including his out-of-school teaching; the many textbooks he wrote for students; articles that he published in the pedagogy journal he co-founded; and the explicitly educational essays he published. The first of these latter educational essays – 'On Instruction in the Classics and the Mathematico-Physical Sciences' (Mach 1886/1986) - is his most systematic treatment of education in general and science education in particular.[2] His other pedagogical papers are: 'On Instruction in Heat Theory' (Mach 1887), and 'On the Psychological and Logical Moment in Scientific Instruction' (Mach 1890/2016), in volumes one and four respectively of his *Zeitschrift*. The last paper is his most detailed account of how psychologically-informed pedagogy should proceed.

Mach's Textbooks

Mach the physicist and philosopher wrote eight university textbooks that went through 20 editions, and many translations.[3] These numerous and much-used textbooks reveal a good deal of his understanding of education as they all had their origin as texts for courses he was teaching. His Prague teaching (1867-95) was mainly to medical and pharmaceutical students. He took an historical approach to all topics in his experimental physics courses; this same approach was accordingly mirrored in the popular textbooks that he wrote on the course topics: *Compendium of Physics for Medical Students* (1863), *Optical-Acoustical Investigations* (1873), *Doppler's Theory of Tone and Colour Change* (1874) and *Theory of Motor Sensations* (1875). In 1891 he published two university physics textbooks. And his popular historical studies - Energy (Mach 1872/1911), Mechanics (Mach 1893/1974), Heat (Mach 1900/1986), Optics (Mach 1926/1953), and essays on mathematics (Mach 1903/1943) - were used by both university students and faculty. The texts provided a psychologically-based and philosophically-informed historical introduction to the different fields of science.

Given the number of textbooks Mach wrote and their wide adoption in German lands, it is not surprising that most of the major European contributors to late nineteenth and early twentieth-century physics mention their debt to his texts. In Einstein's memorable words:

> The fact is that Mach through his historical and critical writings in which he followed the development of the individual sciences with so much love and traced historical details into the inner sanctum of the brain [*Gehirnstübchens*] of path-breaking scientists has had a great influence on our generation of natural scientists. I even believe that the people who consider themselves opponents of Mach, scarcely know how much of Mach's way of thinking they have absorbed, so to say, with their mother's milk. (Einstein 1916/1992, pp.154-55)

Machian Education

Although Mach did not bequeath any systematic and detailed 'theory of education' or any 'pedagogy manual' nevertheless his textbooks, educational essays and reports of his teaching, suffice to identify features of a 'Machian' approach to teaching and learning. These features are: a historically-informed or 'heuristic' method; experiential and manipulative ('hands-on') learning; thought experimentation; liberal, coherent and limited curriculum; elaboration of philosophical content; directly or indirectly teaching the nature of science.

(1) Historical or Heuristic Method

The most notable feature of Machian education is that history informs the curricula and classroom pedagogy. This commitment to history arose from Mach's own early investigations of psycho-physics and sensory physiology. In an autobiographical essay, he comments that:

> The teaching itself led [me] to the opinion that the historical presentation of material was the simplest and the most understandable. Such general conceptual connections revealed the economic motives of cognitive theory and the conception of science as part of a general phenomenon of life and development, a view which finally rounded out [my] biological and economic epistemology. ... Since then I have remained as loyal as possible to the historical way of proceeding [*Gange*] in giving lectures. (Mach 1913/1992, pp.24-25)

Pierre Duhem followed Mach in this regard:

> The legitimate, sure, and fruitful method of preparing a student to receive a physical hypothesis is the historical method . . . that is the best way, surely even the only way, to give those studying physics a correct and clear view of the very complex and living organisation of this science. (Duhem 1906/1954, p.268)

For Mach, the 'historical method' did not mean teaching the history of science; rather he advocated thoughtful, laboratory-based teaching and demonstrations, with students engaged in solving problems and puzzles. This was a form of Inquiry Teaching, but it was guided inquiry with history providing the background or puzzles for inquiry; children's natural curiosity about nature – shadows, tides, magnification, animal behaviour, and so on – can all be connected to people and episodes in the history of science.

Mach was the first to identify educational and social benefits of the historical approach to science teaching. Such benefits that have repeatedly been appealed to by educators, and they can be listed as:[4]

(i) *Easy beginnings*. Replicating the historical progression of a science means that subject matters start simple, are manageable and can to some degree be grasped. The simple pendulum before compound pendulum; the composition of air before photosynthesis; Volta's pile before lithium batteries, etc.
(ii) *Practice in theorising*. Student theorising or hypothesising about historical material or phenomena that they themselves can experience, and then testing these conjectures in practice, can show the centrality of theorising for science. What factors affect the period of a pendulum? Is it sunlight or heat or both that are necessary for photosynthesis?
(iii) *Connectivity*. Following through historical thread of developments in a subject or about a topic can show the interconnection of science with philosophical, social, cultural and religious traditions. The dependence of pendulum physics first on geometry, then calculus; the utilisation of pendulum movement in timekeeping and so solving the longitude problem, etc.
(iv) *Epistemological development*. Over time and with different content, historical approaches can contribute to a more sophisticated scientific, personal and general epistemology; students can become familiar with the basic epistemological questions of: how do we know that some claim is true, and how do we judge one claim as more justified than another? This competence is important inside and outside of the classroom. How do 'accidents' or 'impediments' bear on claims about laws of pendulum motion?
(v) *Sense of tradition*. The historical approach when repeated from subject to subject, and topic to topic, gives students a sense and appreciation of science as a living social tradition that is in lively interaction with its culture and society; and in which each generation or epoch benefits from those that preceded it.

Optics as an Example of Heuristic Teaching

Mach's final major work was his book *The Principles of Physical Optics: An Historical and Philosophical Treatment* (Mach 1926/1953). It is an immensely detailed 320-page book with 279 diagrams and scores of footnotes and citations beginning with Euclid in Greek and working through every major contributor - in Latin, French, German and English - to the science of optics. The material had been the basis of his lectures and experiments in the first year of appointment in Prague (1867). The manuscript and Prefaces were largely completed by 1913 but on account of the outbreak of war, and the further deterioration in his own already crushing and debilitating medical condition (Blackmore 1972, p.180), finalisation and publication did not happen till after his death in 1916, with the English translation appearing in 1926.

The purposes of this chapter are well served by drawing attention to a 1926 review of the book's English translation that made special mention of its treatment of polarisation:

> A feature of the book that deserves special praise is its treatment of polarization and double refraction. .... the present writer has encouraged optics students to begin the study of these subjects with the simplest apparatus and natural crystals, such as were available to the early workers. In this way they obtain a thorough acquaintance with the principal features of the subject ... thus by easy stages the properties of uniaxial crystals become familiar ..[The book] in addition describes many beautiful experiments which will be new to the majority of readers. (Martin 1926/1992, p.70)

Mach devotes two chapters to polarisation, chapters X and XII (60 pages), and he notes:

> As we have seen, great difficulties had to be overcome before the nature of polarization could be thoroughly explained. This was, no doubt, the reason why workers of such renown worked on this problem, and the cause of the tardy nature of its solution. It would thus be worth our while to go into these difficulties. (Mach 1926/1953, p.204)

His university teaching of the topic is guided by the memory of how hopeless was his school introduction to the subject:

> It was while attending grammar school (gymnasium) that I first heard of the transverse vibrations of which light must consist. This view made a very strange, phantastic, and unsympathetic impression on me, without my knowing the actual cause. When I tried, with the aid of memory, to obtain a clearer view of the matter, I had to confess that I felt instinctively the impossibility of transverse vibrations in so readily movable (displaceable) a medium as air, and thus more so for the ether, which I considered must be more rarefied and easily displaceable. (Mach 1926/1953, pp.204-05)

For Mach it was by experimentally working through debates, experiments and achievements in the history of the subject that the 'strange, phantastic, and unsympathetic impressions' were rendered intelligible. He begins with Erasmus Bartholinus' 1670 description, in Latin, of the rotating double images of objects seen through Icelandic spar. And turns then to Huygens account, twenty years later in French, of the polarisation phenomena in his *Treatise on Light* (Huygens 1690/1945). Predictably Mach says that Newton makes progress beyond Huygens because he:

> takes care to proceed along the lines of the views expressed at the beginning of the *Optics*. In experimental science, namely, little regard is to be paid to hypotheses, and the invention of obscure qualities for each phenomenon in particular is not to be encouraged. (Mach 1926/1953, p.189)

Where Huygens held to light rays being uniform, Newton allowed them to be asymmetrical; the ray properties in one direction were different from those in the ray perpendicular to it. By apt and simple manipulation of light through different adjacent 'Island' crystals, Newton showed that the polarisation property was in the light prior to its entry into the crystal, it was not manufactured by the crystal but rather selected by the crystal (Newton 1730/1979, pp. 354-366).

Through the 60 pages, Mach continues his account of the experimental history of polarisation, identifying the new experimentally-manipulated phenomena and the hypotheses and their refinements tested against the phenomena. He draws a typically Machian methodological lesson:

> It is one question, what actual property of light it is that makes itself evident in polarization, and quite a different one, whether this property may be (mechanically) explained or further reduced. The fact that these two questions have not always been clearly distinguished has often had a retarding action on the progress of optics. (Mach 1926/1953, p.205)

(2) Experiential and Experimental Teaching

Although a pre-eminent theorist, and concerned with economy of thought in education, Mach firmly believed that abstractions in the science classroom should, as Hegel said of philosophy, take flight only at dusk: 'Young students should not be spoiled by premature abstraction, but should be made acquainted with their material from living pictures of it before they are made to work with it by purely ratiocinative methods' (Mach 1886/1986, p. 4). Mach constantly returned to this basic defect of pedagogy: the confusing of the logical structure of a discipline with the structuring of its pedagogical presentation. In his lectures in *Space and Geometry* he observes:

> Great inquirers, even in recent times, have been misled into following Euclid's example in the presentation of the results of their inquiries, and so into actually concealing their methods of investigation to the great detriment of science. (Mach 1903/1943, p.113)

Mach develops this point at length in his 'Psychological and Logical Moment' essay published in his co-edited pedagogy journal *Zeitschrift für den physikalischen und chemischen Unterricht* (*Journal of Instruction in Physics and Chemistry*, 1890, vol.4 pp.1-5)[5] where he says:

> The generally prevalent overrating of the logical moment in relation to the psychological – also when one completely disregards the abnormal outgrowth of it completely – meets us often enough. It probably derives from the time when the elements of Euclid were seen as a model of scientific method. (Mach 1890/2016)

Mach is a philosopher and wants students ultimately to appreciate the logical structure of disciplines, meaning the relational structure of concepts, generalisations and laws within disciplines.[6] However Mach's pedagogical point is:

> It nevertheless seems to be clear that the order of the concepts can only occur when and to the extent that concepts are gained in the first place. (Mach 1890/2016)

And this gaining of concepts is a psychological process that begins with and requires tactile and visual experience, not just reading. Putting the logical before the psychological means:

> what at best can be the end of the activity, one wants to begin with. This procedure – the counter-image of the historical way of development – I can only consider as mistaken. (Mach 1890/2016)

Further:

> Also the solution of a problem, if it is to be valued and understood, needs to be prepared psychologically. Even the obscurity, the discrepancy, which comes before the solution of the problem, should be felt. The solution should not appear ready, before the problem has appeared, but rather become gradually. Thereby it also becomes clear that not every produced sentence can and should be indefeasible from the point of view of the resolved problem. Here applies in detail what has been stated about the application of teaching in general. (Mach 1890/2016)

The substitution of words for experience that Mach laments has been a constant source of complaint in science teaching from Mach's time to the present. John Bradley, at the University of Hull is a Mach enthusiast who wrote a PhD thesis and subsequent book on Mach's philosophy of science (Bradley 1971). The book begins:

> My interest in Mach arose out of a lecture on the lever given to freshmen at Cambridge in 1927 by the late Alexander Wood. He advised his students to read Mach, and I have been doing so ever since. (Bradley 1971, p. vii)

Bradley railed against the dominance of theory in school science programmes. The influence of his early reading of Mach is seen in a 1935 article on 'Atomism in the School Certificate' (Bradley 1936). Thirty years later he wrote:

> The young people of this country come hopefully to school asking for the bread of experience; we give them the stones of atomic models. (Bradley 1964, 45, p.366)

His belief was that children should not be exposed to atomic models, or much other theory, until the final school years; theory should be X-rated. The NSF programmes of the USA were in his sights:

> By returning from the far country [USA] with its painted Jezebels of atomic models to the homeland and pure gospel of Armstrong, the teaching of chemistry could be immensely improved without the expenditure of a penny. Indeed money could be saved, because sulphuric acid is cheaper than models of models of models. (Bradley 1964, 45, p. 366)

(3) Thought Experiments

A special feature of Mach's view of science education was his advocacy of thought experimentation (*Gedankenexperimente*).[7] He said of this that 'Experimenting in thought is important not only for the professional inquirer, but also for mental development as such', not only the student but 'the teacher gains immeasurably by this method' (Mach 1896/1976, p.143). Thought experiments enabled the teacher to know what grasp students had on the fundamental concepts of a discipline. At a simple level 'The method of letting people guess the outcome of an experimental arrangement has didactic value too' (Mach 1905/1976, p.142).

It is noteworthy that each edition of his *Zeitschrift* carried thought experiments for his readers to perform. For instance, he asks, what is expected to happen to a beaker of water in equilibrium on a balance when a suspended mass is lowered into it? Or in another issue, what happens when a stoppered bottle with a fly on its base is in equilibrium on a balance and then the fly takes off? These examples are of thought experiments of an anticipatory type: the actual experiment can be performed. Mach wanted such thought experimentation to be part and parcel of physics education. The 'experiments' engage the mind, and they reveal what a student believes about the relevant concepts being investigated. He believed that the exercise of creative imagination was another way of bridging the gap between humanities and the sciences: 'The planner, the builder of castles in the air, the novelist, the author of social and technological utopias is experimenting with thought' (Mach 1896/1976, p. 136). Their utilisation does require that teachers know about them; that teachers have some training in the history and philosophy of the subject they are teaching.

Thought experiments were not only important for pedagogy. They were not merely a way of finding out about students' knowledge and grasp of concepts; rather they latched onto a fundamental feature of science: the centrality of idealisation and abstraction in scientific practice. Surveying the history of science, Mach notes how often experimenters have to refine procedures and apparatus so as to minimise or remove 'impediments' and 'accidents', to use Galileo's terminology.[8] He notes that:

> Physically, such a process is often impossible to carry out, so that we may speak of it as an idealization or abstraction. (Mach 1896/1976, p.140)

And goes on to say that Newton's law of inertia was discovered by abstraction, so too Kirchhoff's notion of the perfect black body, Carnot's absolute insulator, and most other laws of physics. Further:

> All general physical concepts and laws, the concept of a ray, the laws of dioptics, Boyle's law and so on are obtained by idealization (Mach 1896/1976, p.140)

Mach's advocacy of thought experiment did not take the education world by storm. In the late nineteenth century, science was not much taught, and where it was, imagination, hypothesizing and creative thought were not much valued. Einstein, who was to place thought experiments upon the centre stage of modern physics, made the oft-quoted remark about his own schooling that: 'after I passed the final examination, I found the consideration of any scientific problems distasteful to me for an entire year', and 'It is, in fact, nothing short of a miracle that the modern methods of instruction have not entirely strangled the holy curiosity of inquiry' (Schilpp 1951, p.17). Einstein of course thanked Mach for opening his mind to the possibility of novel ways to conceptualise physics and for 'legitimising' creative thought in science. Concerning his special theory of relativity, he says:

> The type of critical reasoning which was required for the discovery of this central point was decisively furthered, in my case, especially by the reading of . . . Ernst Mach's philosophical writings. (Schilpp 1951, p.53)

(4) Liberal Curricula

Mach refused to identify 'liberal education' with 'classical education'. He said the latter, in part, needs to meet 'the general wants of the times', and clearly the classical curriculum did not do so (Mach 1886/1986, p.371). Consequently: 'A truly liberal education is unquestionably very rare' (Mach 1886/1986, p.371). And further:

> The *schools* can hardly offer such; at best they can only bring home to the student the necessity of it. It is, then, his business to acquire, as best he can, a more or less liberal education. (ibid.)

Machian science curricula are found in present-day Liberal Education programmes where specialisation is avoided, intra-disciplinary and inter-disciplinary connections are stressed, philosophical content is developed, ethical and moral dimensions are high-lighted, and so on.[9] Mach thought a thinned-out curriculum, taught in what might now be called a historical-investigative or heuristic manner, gave the best chance of realising a satisfactory liberal education:

> every young student could come into living contact with and pursue to their ultimate logical consequences merely a *few* mathematical or scientific discoveries. Such selections would be mainly and naturally associated with selections from the great scientific classics. A few powerful and lucid ideas could thus be made to take root in the mind and receive thorough elaboration. (Mach 1886/1986, p.368)

Using reference material from the web and well-documented texts this Machian goal is reachable.[10]

Mach saw bloated curricula as an obstacle to education. For him the central aims of education were to promote knowledge and understanding of appropriate topics across a range of disciplines, to improve reasoning, to encourage imagination, and to advance autonomous or independent thinking. An over-stuffed curriculum, and much more so, teaching for the test of such a curriculum, frustrated these aims:

> I know nothing more terrible than the poor creatures who have learned too much. What they have acquired is a spider's web of thoughts too weak to furnish sure supports, but complicated enough to produce confusion. ... It is not necessary that all the matter that is offered in schools should be learned. ... How can the mind thrive when matter is heaped on matter, and new materials piled constantly on old, undigested materials? (Mach 1886/1986, p.367)

And for Mach:

> It seems also unnecessary that all branches should be treated at school, and that exactly the same studies should be pursued in all schools. ... Uniforms are excellent for soldiers, but they will not fit heads ... A certain amount of liberty in the choice of studies must be introduced in the upper classes (Mach 1886/1986, pp.367, 369, 370)

One hundred years later these laments are still being voiced about the US 'one mile wide and one inch deep' curricula. The present telephone-directory sized *Next Generation Science Standards* (NRC 2013) with all its sequenced 'cross-cutting' concepts, embedded in 'The Common Core' with national, uniform high-stakes testing - does nothing to abate this concern, only strengthens it.

Mach was writing about late nineteenth-century Austrian education, which was dramatically unequal, illiberal, hidebound, clerically dominated and verging on fossilized. But just as he refused to grant 'timelessness' to metaphysics in science, it would be un-Machian to just carry over Mach's diagnoses and remedies for his own time to contemporary education. Yet his solution to the educational ills of his period was democratic and not without current merit:

> But the path is marked out for us; the will of the people must acquire and exert upon our school legislation a greater and more powerful influence. Furthermore, the questions at issue must be publicly and candidly discussed that the views of the people may be clarified. (Mach 1886/1986, pp. 374)

(5) Philosophy in Science Teaching

For Mach, the teaching of science, or any discipline at all, went hand-in-hand with teaching the philosophy of the discipline. For a student to understand the discipline and its claims, they had to appreciate the methodology, epistemology, ontology and related ethics and goals of the discipline; know how it came to make its claims and how these claims were substantiated; to appreciate the role of internal and external factors in the process of substantiation. Acquiring such philosophical understanding of a discipline meant attending to its history. And this held whether the discipline was science, mathematics, economics, psychology, history, theology or anything else. An example of Mach's 'philosophy of the discipline ' concern is:

> I led [during doctoral examinations in Vienna, 1895-1898] candidates into a conversation on general, and even the most general, questions of their special field. I recommended to philologists that they study the writings of philosophers of speech, to historians cultural history and prehistory, and mathematicians and natural scientists normally Mill and Jevons. It often became evident that the candidates did not know the philosophical writings of their own special fields. They were usually very thankful for my suggestions about future study. (Blackmore 1972, p.139)

For Mach, philosophy was in the weft and warp of science (and indeed of all subjects being taught). Philosophical reflection does not have to be imported into science teaching, it is already there in the textbook, laboratory and classroom; it just needs to be recognised and elaborated. Otto Blüh, the Machian refugee, physicist and pedagogue, stated this well in the Preface to his physics textbook:

> This book further offers the student the opportunity of becoming acquainted with the historical and cultural relations of physics, in the belief that the education of a scientist can be advanced most effectively by giving a significant place to the philosophical, social, and moral implication of physical science *within* the physics curriculum rather than through supplementary so-called humanistic and social studies. Such course work or reading, valuable in itself, will make its mark on the scientist's intellectual and cultural development only if it is intimately related to his scientific studies proper. (Blüh & Elder 1955, p.vii)

At a most basic level any text or scientific discussion will contain terms such as 'law', 'theory', 'model', 'explanation', 'causation,' 'truth,' 'knowledge', 'hypothesis', 'confirmation', 'observation', 'evidence', 'refutation', 'idealisation', 'time', 'space', 'fields', 'species', 'proof', 'evidence', 'mass', and so on. Philosophy begins when students and teachers slow down the science lesson and ask what these terms mean and what the conditions are for their correct use. Students and teachers can be encouraged to ask the philosopher's standard questions: What do you mean by ____? And, how do you know ___? of all these concepts. The chapters in any 'Introduction to Philosophy of Science' textbook will all deal with all of these topics and teachers should be encouraged, indeed they should feel obligated, to read and engage with such texts.

Philosophy is a part of all of science, and this is especially obvious in Newtonian theory, a staple of all science classrooms.[11] Mach had the greatest respect for the genius of Newton, for his 'intellectual greatness' (Mach 1893/1974, p.304), and excused his failure to deeply appraise the foundations of his 'system of the world' because: 'He that has to acquire a new point of view naturally cannot possess it so securely from the beginning as they that receive it unlaboriously from him' (Mach 1893/1974, pp.304-05). But Mach says that after two hundred years the situation is different and Newton might well have expected those following him to more closely attend to, scrutinize, and philosophise about the foundations of the system they were 'unlaboriously' receiving.

This is not just a task for philosophers; Mach saw that the task can begin in science classrooms whenever the Newtonian system (or Einsteinian, Darwinian, Mendelian) is taught. But this rarely happens. Herbert Goldstein, in his popular *Classical Mechanics* book, lays out the standard procedure:

> Basic to any presentation of mechanics are a number of fundamental physical concepts, such as space, time, simultaneity, mass, and force. ... For the most part, however, these concepts will not be analysed critically here; rather, they will be assumed as undefined terms whose meanings are familiar to the reader. (Goldstein 1950/1980, in Assis & Zylberstajn 2010, p.143)

Mach might say 'familiar, but not understood'; and further would note the missed opportunity to encourage students to put their toe in the philosophical water. The opportunity for basic philosophical engagement is everywhere in science, but is everywhere put off - 'later, later, later'. At best this deferment goes on to postgraduate years, but usually 'later' does not come even then.

Mach's view on the place of philosophy in science education is clearly seen in the writings and teaching of Philipp Frank, the Viennese physicist who studied in Mach's department, who often expressed his great debt to Mach, and who was a founding member of the 'Mach Circle' in Vienna, and was one of the foundational Positivists in 20th century philosophy. He is an instantiation of Machian educational ideals.[12] Peter Bergmann went to Vienna in 1933 as an 18 year-old Jewish refugee from Berlin, and subsequently recalled how:

> In this overheated and jittery atmosphere there was one fatherly figure who represented all that was best at the University, Philipp Frank. … He would encourage all of us students, and he gave us the feeling of a wide-open intellectual window, open to things that happened in and out of physics, and open to things that happened outside of the country as well. Philipp Frank saw to it that there was close contact with philosophy of science … with experimental physics … and with pure mathematics. (Blackmore, Itagaki & Tanaka 2001, p.69)

Twenty years later, Jeremy Bernstein gave the following account of Frank's Harvard classes:

> Professor Frank spoke to us – 'lecture' would be too formal a term – for about an hour .. followed by a second hour of discussion. Nothing pleased him more than sharp disagreement with his own points of view in these discussions …. I think that all of us who attended these classes were constantly awed, although this was never Professor Frank's intention, by his almost incredible erudition. He seemed to have read and digested the great philosophical, literary and scientific works in an enormous variety of languages. He once told me that he had studied Arabic, as a young student, in order to be able to read the great texts in that language. … This vast general culture was also worn instinctively, without pretense, and with the same mastery that characterized his scientific cultivation. (Blackmore, Itagaki & Tanaka 2001, p.71)

These complimentary reflections of Frank, perhaps not surprisingly, mirror those of Mach made by William James in 1882 and cited at the beginning of the chapter.

Mach knew that science developed in conjunction with philosophy, being both influenced by it and in turn influencing it; thus all his historical studies illuminated this connection.[13] Frank was more explicit about its educational consequences, saying:

> Equally, students of science and philosophy should learn exactly what were the issues between Descartes and Newton, and between Newton and Leibnitz. From these disputes has arisen what we now call the classical physics of the nineteenth century, which until today has been the basis of the training in science [required] to get into colleges of engineering or liberal arts. To grasp these issues would help them to understand our present science as a dynamic living being. (Frank 1950, pp.279-280)

Such broad, philosophically-informed teaching enables students to appreciate the engagement of science with philosophical systems, religion and political ideology. For Frank:

> There is no better way to understand the philosophic basis of political and religious creeds than by their connection with science ... the influence of political and religious trends on the choice of these symbols [metaphysics of science] should by no means be minimized, as is often done in presentation of the philosophy of science. (Frank 1950, p.281)

It barely needs stating that these accounts of Frank's pedagogy are at odds with the view of positivists as dogmatic, over-bearing, pupil-ignoring, adherents of the 'banking' or 'fill-them-up' view of teaching so popular in education circles. [14]

Nearly 50 years ago Israel Scheffler outlined the contribution that philosophy can make to education, and did so in terms that echoed much of what Mach had written:

> I have outlined four main efforts through which philosophies-of might contribute to education: (1) the analytic description of forms of thought represented by teaching subjects; (2) the evaluation and criticism of such forms of thought; (3) the analysis of specific materials so as to systematize and exhibit them as exemplifications of forms of thought; and (4) the interpretation of particular exemplifications in terms accessible to the novice. (Scheffler 1973, p.40)

Comparable things were stated in a 1981 review of the place of philosophy of science in British science-teacher education:

> This more philosophical background which is being advocated for teachers would, it is believed, enable them to handle their science teaching in a more informed and versatile manner and to be in a more effective position to help their pupils build up the coherent picture of science – appropriate to age and ability – which is so often lacking. (Manuel 1981, p.771)

In recent decades there has been a good deal of writing and research on the contribution of history and philosophy of science (HPS) to science teaching.[15] One part of this contribution has been the recognition of the connection of science to other academic and cultural fields. This is constantly pointed to in Mach's historical works. One part of the contribution of HPS to teachers' and educators' understanding is to connect topics within particular scientific disciplines; to connect the disciplines of science with each other; to connect the sciences generally with other disciplines such as mathematics, philosophy, literature, psychology, history, technology, economics, and theology; and finally, to display the interconnections between science and components of culture – the arts, ethics, religion, politics. All of this is obvious in Mach's work, and is developed in more detail by Frank and other contributors to the Machian tradition.

At the same time there has been a concerted effort by researchers and curriculum writers to include Nature of Science (NOS) in science programmes.[16] One problem has been that because HPS is so little taught in graduate education programmes, this NOS research frequently underestimates the complexity of the HPS issues and debates, and presents a too simplified account of the issues. One especially deleterious effect of this underestimation, the more so when it becomes hubris,[17] is the presenting of deeply controversial issues in HPS as settled, and so producing numbered lists that purport to capture the nature of science, and going on to have such lists taught catechism-like in school classes and teacher-education programmes. The learning of such lists benefits no one, especially when they appear in national and provincial curricula and become required learning for high-stakes exams.[18]

Conclusion

Mach's Enlightenment-informed approach to education, characterised by Reformism, knowledge-seeking, Experientialism and Liberalism can be championed without commitment to his Phenomenalism. The last was the core of all his scientific and philosophical work, yet his theory of education and his pedagogical style can survive without it. Mach's pedagogical advice was fairly simple:

* Begin instruction with concrete materials and familiarize students with the phenomena discussed.
* Teach a little, but teach it well.
* Be guided by the historical order of development of a subject.
* Aim for understanding and comprehension of the subject matter.
* Tailor teaching to the intellectual level and capacity of students.
* Address the philosophical dimensions and questions that arise in all science teaching.
* Show that just as individual ideas can be improved, so also scientific ideas have constantly been, and will continue to be, overhauled and improved.
* Engage the mind of the learner.

But each needs, in the present time, the kind of historical, philosophical and educational elaboration, and where necessary qualification, that Mach provided for his own late nineteenth-century Austrian circumstance.

Being a Realist is no bar to being a Machian in either the theory or practice of education. But thoughtful realism does require coming to terms with Mach's own phenomenalist arguments that he so comprehensively advanced. Science teachers have endless opportunity to do this. Whenever 'attraction at a distance', 'magnetic fields', 'electron shells', 'inertial mass', 'atomic models' are mentioned, aspects of the phenomenalist/realist debate can be introduced. So much else of what Mach values as philosophy, and that is so much a part of science – good experimental design, logical thinking, justified connection of evidence to conclusions, values - is quite independent of his phenomenalism and warrants attention by teachers.

A great pity is that there is so little opportunity in any country's science teacher-education programmes, or even graduate education programmes, to learn from the life, work and writings of Mach. The history and philosophy of science is not a part of these programmes and neither is the history of science education. With both HPS and history of education missing, Mach does not appear in pre-service or graduate education programmes that are dominated by other supposedly more practical concerns. But in education, as in science, there is nothing so practical as a good theory, and Mach provides one. His theory can in principle be elaborated, revised and criticised. It is an orientation and guide to education and pedagogy that rewards engagement.

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[1] Direct dependency means ensuring enrolments in university undergraduate and graduate science programmes; indirect dependency means support of the general population for funding and provision of such research. James Conant was forthright in arguing these dependencies in the USA after the Second War (Conant 1947).

[2] The essay was originally prepared for a 1881 conference of Austrian scientists discussing the subject of science instruction in schools. It was subsequently expanded and delivered before the 1886 Congress of Delegates of German College Teachers. Mach took the occasion to decry the stranglehold of Classics on university entrance, and to argue that the lauded cultural and humanistic goals of Classical education can equally be gained by good historically and philosophically informed science education.

[3] For publication details and history of these textbooks, see Blackmore, Itagaki & Tanaka (2001b).

[4] Arguments and evidences for an historically informed and grounded curriculum and pedagogy are outlined in Matthews (2015, chap.4).

[5] The journal is available on the web. It was the second ever science pedagogy journal, the first being *Zeitschrift für mathematischen und naturwissenschaflichen Unterricht* which began publication in 1870 and was edited by J.C.V. Hoffmann, a secondary school teacher in the Saxony mining town of Freiberg. (Thanks to Kathryn Olesko for this information.)

[6] He of course says that a discipline has no fixed, immutable structure; with the choice of different fundaments or primitives, a new disciplinary structure is created, but nevertheless this also has a conceptual structure which ultimately needs be grasped.

[7] Mach's views on thought experiments are most thoroughly developed in *Knowledge and Error* (Mach 1905/1976, chap.XI).

[8] On Galileo and idealization see Koertge (1977) and McMullin (1985).

[9] There is a large literature on the theory and practice of liberal education, sometimes called 'general' or 'humanistic' education. See at least Bantock (1981, chap. 4) Peters (1966, chaps.1, 2), Schwab (1949/1978, 1950/1978) and contributions to Kirby & van der Wende (2016).

[10] For texts see Holton (1952/2001). For historical texts and experiments that can be utilised in classes, see Kuehn (2014, 2015). For the philosophical contexts and implications of these episodes in the history of science see Weinert (2005).

[11] Russell Norwood Hanson (1965) provides a nice exposition of this point; Arnold Arons (1988) gives an indication of what physics teachers can do about the situation.

[12] Some of Frank's education essays are Frank (1950b, 1950c). For the life and publications of Frank see Blackmore, Itagaki & Tanaka (2001, chap.3) and Stadler (2001, pp.631-36); also Thomas Kuhn's interview of Frank (Frank 1962/2001).

[13] There are numerous works on the interdependence of science and philosophy. See at least d'Espagnat (2006), Trusted (1991), Wartofsky (1968) and Weinert (2005). Some texts and commentary are provided in Matthews (1989).

[14] The disjunction between the reality of positivist-inspired pedagogy and its current educational image is discussed in Matthews (2004, 2015 pp.43-45).

[15] See at least Matthews (2015) and contributions to the three-volume Matthews (2014).

[16] For guides to the arguments and literature see Erduran & Dagher (2014) and Hodson (2014).

[17] One egregious example is the causal announcement that 'constructivism is the most mature epistemological commitment' (Roth & Roychoudhury 1994, p.28). This labels at least half of the international philosophy of science community, the 'realist' half, immature.

[18] Having modest goals for NOS teaching is advocated in Matthews (1998). The benefits of moving educational discussion from nature of science to features of science are outlined in Matthews (2011) where it is argued that the latter formulation invites discussion and elaboration of multiple features of science in the way that NOS terminology, and associated assessment, does not.

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