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Mathematics in Science: The Role of the History of Science in Communicating the Significance of Mathematical Formalism in Science

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ABSTRACT: The use of a historical profile for illustrating the significance of the mathematical components of a scientific law is discussed in this paper. Such an approach addresses the need for the purposive use of scientific laws rather than the blind substitutionary procedures characteristic of most problem-solvers. The approach has the potential for increasing female participation in the physical sciences because of its reliance on learning modes favourable to female participation.

INTRODUCTION

The use of mathematics in school science has received critical attention from science education researchers and philosophers of education and science over the last decade. Gabel, Sherwood and Enochs have examined the problem-solving strategies used by high school chemistry students in Indiana (U.S.A) and concluded that, 'the majority of students solved the chemistry problems using only algorithmic methods, and did not understand the chemical concepts on which the problems were based' (Gabel *et al.* 1984, p. 221). Reif, on reviewing common student behaviours in solving problems in basic college-level science courses, says that,

> Students tend to place a great emphasis on remembering and using facts and mathematical formulas, without trying to embed these in a rich framework of qualitative knowledge. Accordingly, students may be able to answer some quantitative questions by merely manipulating some mathematical formulas, but they may be quite unable to answer simple qualitative questions of a similar kind. Furthermore, students seldom use qualitative knowledge to plan solutions or to check whether results obtained by them make any sense. (Reif 1983, p. 952)

David Stenhouse, a philosopher of science and education, argues that,

> What is inculcated in a great deal of science education is, all too often, not understanding, but rather a sort of recipe book acquaintance with a number of mathematical techniques which tend often to be used on an ad-hoc basis without any proper understanding either of the mathematical theory and assumptions on which the techniques are based, or of the subject matter to which they are applied. (Stenhouse 1985, p. 21)

It has been suggested (Ormerod *et al.* 1979) that such a quantitative approach as outlined above may be responsible for the differential achievement of males and females in physical science (Piburn and Baker 1989; Haggerty 1987). However a quantitative approach which embeds the mathematical concepts in a social context (Harding 1983) and which adopts a cooperative learning mode rather than an individual competitive learning mode (Owen and Barnes 1982; Harding 1983) is likely to enhance female participation and performance in the physical sciences.

Some approaches that have been adopted to overcome a mathematical approach which represents a blind manipulation of symbols without any understanding of the concepts involved have been as follows.

1. An approach which gives an emphasis to the qualitative framework of the concepts involved in a mathematical equation. This approach has been championed by Reif (1983) for problems in mechanics by way of what he terms 'ancillary knowledge'. It is the knowledge required to,

> specify the concept or principle by descriptive statements and by detailed procedures needed to identify the concept; it includes the knowledge needed to apply the concept in various kinds of instances; and it includes explicit warnings about likely errors and the requisite knowledge to discriminate them from correct situations. The ancillary knowledge includes also familiarity with some basic implications and applications of a concept or principle, as well as explicit guidelines specifying the conditions when the concept is likely to be useful. (Reif 1983, p. 951)

De Berg (1990b) has adopted this approach in developing qualitative or semiquantitative exercises to be used in conjunction with the pressure-volume law for gases (Boyle's Law).

2. An approach which emphasizes model development. Lind (1980), in discussing the model character of physical conceptions, suggests,

> If the development of models is to play a central role in physics education, mathematization should not set in too early. Where mathematization is attempted, it should be for the purpose of model development. In this way, the danger of losing sight of the physical background of mathematical expressions is probably lessened: there is a need to interpret what has been derived in mathematical terms; also, problems will first be discussed with reference to models and so pupils will be less inclined to apply mathematical formalisms mechanically. (Lind 1980)

3. An approach which uses physical materials. Wollman and Lawson (1978) have shown that individualised procedures based on the use of physical materials are more effective than verbal textbook procedures in teaching students to solve problems involving the formal scheme of proportionality. They comment that, 'relatively little is gained from the use of algorithms without first establishing a conceptual groundwork that would motivate their construction' (Wollman and Lawson 1978, p. 231). The approach making use of physical materials, 'while not as coherent logically nor as straightforward pedagogically as in rule instruction is more in keeping with spontaneous growth as observed in both individual and historical contexts' (Wollman and Lawson 1978, p. 231).

In this article I wish to outline an approach which I have developed for communicating the significance of mathematical equations in science which makes use of a historical profile developed from primary sources relating to the pressure-volume law for gases. However, we should look first at some of the background of the role of mathematics in science to provide a setting for our discussion.

ROLE OF MATHEMATICS IN SCIENCE

The first mathematical model of nature is thought to have been given us in the modern era by Galileo Galilei (1564–1642). This is the view expressed by such prominent writers as Morris Kline (1953), Philip Jourdain (1956) and P.M. Harman (1983). Harman comments that,

> In place of Aristotle's explanation of the centrality of the earth in the universe as a consequence of a heavy 'earthy' body moving to its natural place, Galileo aimed to create a theory of nature in which physical problems were conceived in mathematical terms —. While Aristotle regarded mathematics as inapplicable to the real physical world, Galileo sought to create a physics which was based on mathematization. (Harman 1983, p. 19)

In shifting the emphasis to the 'how' rather than the 'why' with respect to the occurrences of nature, Galileo was able to demonstrate, for example, that the fall of bodies to the earth could be expressed mathematically in terms of a relation between the distance and time of fall, not merely in terms of the tendency of bodies to move to their natural place as suggested by Aristotle. Galileo arrived at a mathematical description of this process, firstly, by geometric constructions based on his definition of uniformly accelerated motion as (Galileo 1638, p. 203), 'A motion such that when starting from rest, its momentum receives equal increments in equal times'. Galileo (1638, p. 200) defined it this way because he believed 'in following the habit and custom of nature herself — to employ only those means which are most common, simple and easy'. When his geometric constructions revealed the proposition that 'The spaces described by a body falling from rest with a uniformly accelerated motion are to each other as the squares of the time intervals employed in traversing these distances', (Galileo 1638, p. 206), it was then left to provide experimental justification for this relationship in nature, a task Galileo describes in some detail in his book, *The Two New Sciences* (Galileo 1638, p. 208). Kline says of Galileo's work that, 'By ignoring friction and air resistance and by imagining motion to take place in a pure Euclidean vacuum he discovered the correct fundamental principle. His trick was to geometrize the problem and then obtain the law' (1953, p. 221). Not every quantification emergence in science follows Galileo's pattern but the principal thrust remains the same; that is, to seek the mathematical formulas that describe nature's behaviour. According to Kline 'such formulas have proved to be the most valuable knowledge man has ever acquired about nature' (1953, p. 216).

Norman Robert Campbell establishes three uses for mathematics in science.

(a) The establishment of systems of derived measurement. Density as derived from mass and volume via the equation, $d = m/v$, is quoted as an example.

(b) Calculations in the form of combining numerical relations to produce new numerical relations. For example, combining the relations,

$$d_1/d_2 = v_1^2/v_2^2$$

and

$$d_1/d_2 = t_1^2/t_2^2$$

gives

$$v_1/v_2 = t_1/t_2$$

as a new relation.

(c) Formulation of theories. This formulation may be based on analogies with known laws such as the development of the kinetic-molecular theory of gases based on the laws of Galileo and Newton. On the other hand, the formulation could be based on a mathematician's sense of form and symmetry. This was the case in Maxwell's treatment of differential equations representing electrical properties which led to the development of wireless telegraphy. Thus a study of quantification in science will represent a study of the foundations of some of the most important discoveries of science (1921, pp. 222–238).

Nobel prize winner, Eugene Wigner, on reviewing the use of mathematics in science commented that, 'the enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious and — there is no rational explanation for it' (1960, p. 45). Raymond Wilder (1981, p. 45) suggests that the common cultural origins of mathematics and the natural sciences has led to the exquisite usefulness of mathematics to the scientific enterprise. However, Jourdain cautions us to remember that our dynamical world expressed in mathematical terms should 'be called a model of reality and must not be confused with the reality itself' (1956, p. 44). In any case, mathematics and science as partners in the building of knowledge have formed as important a part of our heritage as art, music and sport. On occasions, developments in mathematics have assisted developments in science and developments in science have assisted developments in mathematics such as in the relationship between quantum theory and physics.

USE OF THE HISTORY OF SCIENCE

After an analysis of the treatment given the pressure-volume law for gases in fourteen chemistry and fourteen physics textbooks at high school level, de Berg (1989) concluded that the history of science was practically ignored by the textbooks. Certainly, there were descriptive statements from the history of science but there were no exercises on the pressure-volume law which drew upon the historical development of the law. This is also the case for the other gas laws (de Berg 1988). In a study of the perceptions of 104 teachers in Western Australia and the Northern Territory in relation

TABLE 1
Mean and standard deviation for items related to the use of the history of science in teaching the gas laws.

Item	Mean	Standard Deviation
1. I mention the historical aspects of the development of the gas laws in my teaching.	4.41	2.22
2. I set exam questions on the historical aspects of the gas laws.	1.00	1.49
3. I set assignments and/or projects on the historical aspects of the gas laws.	1.27	1.81
4. I use the historical aspects of the gas laws to encourage discussion/debate with my students.	2.56	2.16

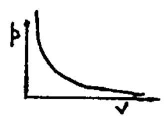
Note. The items were scored on a scale from 0(never) to 8(always).
High values signify agreement with the statement.
Low values signify disagreement with the statement.
104 teachers participated in the study.

to the extent which they used the history of science in their teaching of the gas laws, the teachers indicated only minimal usage of the history of science (de Berg 1988). The teachers in this study were asked to record on a scale from 0 (never) to 8 (always) the extent to which they adhered to a specific statement. The results are shown in Table 1 and indicate that while teachers may be inclined to mention historical aspects of the gas laws on occasions, very little use is made by teachers of the history of science in an instructional sense as revealed by the low scores on the last three items. It was in order to expand the instructional use of history that I set about developing a historical profile of the pressure-volume law in 1986 through an examination of primary source documents.

The historical profile of the pressure-volume law is discussed in detail elsewhere (de Berg 1990) and the profile is reproduced in Table 2 for our reference. It should be noted that while some of the items in Table 2 may not represent historical firsts every attempt was made to locate the origins of a mathematical component of the pressure-volume law. Further historical research may shed further light on the origins of some of these components. In Boyle's and Mariotte's works the pressure-volume relationship is never stated in algebraic symbols. It is only stated verbally in such phrases as, 'pressures and expansions in reciprocal proportion' (Boyle 1662, p. 158), and 'air is condensed in proportion to the weight with which it is loaded' (Mariotte 1676, p. 91). In fact, up to the time of Descartes (1596–1650), and some time after, quantitative relationships were written predominantly in words or in word abbreviations. Flegg (1983) notes that while our modern algebraic notation for quantitative relationships came with Descartes in the seventeenth century, it was not immediately popular

TABLE 2

Historical Profile of the Pressure-Volume Law

Year	Form	Context	Reference
1662	Pressures and Expansions in reciprocal proportions; Boyle, Townley, Hooke, Brouncker.	Data to refute Funicular Hypothesis	Boyle (1662, pp. 156-163)
1676	Air is condensed in proportion to the weight with which it is loaded: Mariotte	In Answer to 'question one may ask'	Mariotte (1676, pp. 88-92)
1768	Proportional Symbol, $\propto$, introduced; Emerson.	Mathematical Notation	Emerson (1768, p. 297)
1849	$p_o v_o$ is constant - Law of Mariotte and Boyle; W. Thomson.	Thermodynamic Study of Steam and Air Engine.	Thomson* (1882, p. 131)
1849	$\frac{dp}{dv} = \frac{-p}{v}$ - Boyle and Mariotte's Law; W. Thomson.	Thermodynamic Study of Steam and Air engine.	Thomson* (1882, p. 204)
1851	$pu = p^1u^1$; W. Thomson.	Thermodynamic Study of Steam and Air Engine.	Thomson* (1882, p. 219)
1902	$pv = c$; $p_1v_1 = p_2v_2$; W. Ostwald. 	Pedagogic - Textbook	Ostwald* (1902, p. 76)
1953	$v \propto \frac{1}{P}$; W.G. Henderson	Pedagogic - Textbook.	Henderson (1953, p. 158)

Note: These references (*) were consulted at the British Museum and Library.

with his mathematical contemporaries. By the eighteenth century, however, the new notation had become the norm for mathematicians throughout Western Europe. Flegg, in reflecting on the reasons for the late adoption of abstract symbols in mathematics, makes the following pertinent comment:

> Mathematical ideas were explained in words; mathematical arguments were written in words. To adopt abbreviations of words is therefore a natural step; the change to abstract symbolism demands an intellectual leap of extraordinary magnitude. (1983, p. 224)

This leap of extraordinary magnitude is witnessed in the forms in which the pressure-volume law has been expressed historically. Although algebraic forms of the pressure-volume law for gases appear as early as in the work of Bernouille (1738) and Waterston (1892), these forms make use of symbols relating to the peculiar model of a gas relating to kinetic theory that is proposed in each case and are not, therefore, included in Table 2. In Table 2, the general algebraic forms of the pressure-volume law used by Lord Kelvin (William Thomson) as early as 1849 are included. The context in each case in which the shown algebraic forms appear is a theoretical, thermodynamic study of the steam and air engines. This study makes extensive use of calculus as shown in the derivative form of the pressure-volume law. The advantages of using the algebraic notation over the verbal notation should be clear at this point. In particular, the generation of new mathematical expressions, such as the expression for dp/dv, provides new information. For example, with the help of this expression it is relatively easy to draw tangents to the P–V curve by drawing a line parallel to the line passing through the P and V values on their respective axes.

A graphical interpretation of the inverse proportion was only made possible after Descartes' introduction of coordinate geometry. Although Waterston (1892, p. 275) uses a pressure-volume graph as early as 1845, a more extensive background discussion of the graphical relationship is given by Wilhelm Ostwald in his *Principles of Inorganic Chemistry* published in 1902. Ostwald explains the usefulness of the graphical technique in the words:

> In many cases, however, especially in the investigation of new relations, an algebraic expression for a really existing dependence is not known. In such cases it is important to possess a method which allows of showing clearly the connection between the magnitudes, so that the general relations can be judged. For this purpose the representation by means of coordinates is generally used in the experimental sciences. (Ostwald 1902, p. 73)

The proportional symbol, $\propto$, whilst introduced by Emerson in 1768, does not appear to be used in the pressure-volume law until this century. The difficulty in accurately extrapolating to new values on the P–V curve should be clear and the use of the P against $1/V$ graph, which is linear, may be seen as a tool for eliminating this problem. This helps students to understand the significance of linearizing relationships in science.

Aspects of quantification, drawn from the historical profile in Table 2, which might prove particularly pertinent to the teaching-learning situation can be summarized thus: – verbal statements precede algebraic statements (as exemplified in Boyle's and Mariotte's statements compared with those of Thomson); qualitative descriptions precede quantitative descriptions (it should be remembered that Boyle's publication of the pressure-volume law came after the publication of the 43 experiments describing the qualitative properties of air); the emergence of a quantitative form may be due to an external need such as the need to refute an alternative hypothesis, or it may be due to an internal, speculative, curiosity-ridden composure; the emergence of the pressure-volume law is dynamic, that is, the emergence profile contains background information, explicit experimental details, information as to how and why the mathematical relationship is determined, and a comment on the accuracy of the relationship. (A reading of Boyle's original account of the experiment originally published in 1662 will confirm this. A static emergence profile is one in which the mathematical relationship is stated without recourse to background information, experimental details, how and why the relationship is derived or the usefulness of the relationship); the portrayal of data graphically can assist in the deduction of the relationship between the variables involved; the importance of precision equipment for making accurate measurements is evident (the carefully constructed air-engine and glass U-tube played a significant part in deducing the pressure-volume law); and the notion that a mathematical description of reality may only be approximate is present (this is evident in Boyle's statements of caution regarding the applicability of the reciprocal proportion for pressure and volume. For example, he says, 'the proportion was suitable enough to what might be expected to allow us to make this reflection upon the whole; that whether or no the intimated theory will hold exactly — for about that, as I said before, I dare determine nothing resolutely till I have further considered the matter' (Boyle 1662, p. 162). Six of these aspects of quantification are illustrated in Figure 1.

In Figure 2, I have listed six of the questions which I use in my first year chemistry class in order to address some of the aspects in Figure 1 drawn from the historical profile of the pressure-volume law. These questions address the historical context and the nature of the mathematical forms as they emerged in time, and avoid the procedure which concentrates only on substitution into the formula, $P_1V_1 = P_2V_2$. The questions encourage discussion and debate and provide an atmosphere in which female students participate more willingly (Harding 1983). In my experience, students think that Robert Boyle expressed the P–V relationship algebraically and graphically and are surprised to learn that he used a verbal mathematical statement only along with tabulated experimental data. This situation arises because students have usually been exposed to erroneous statements in high school textbooks regarding the discovery of Boyle's Law (de Berg 1989, p. 123). The dynamic development of science is lost if only summary

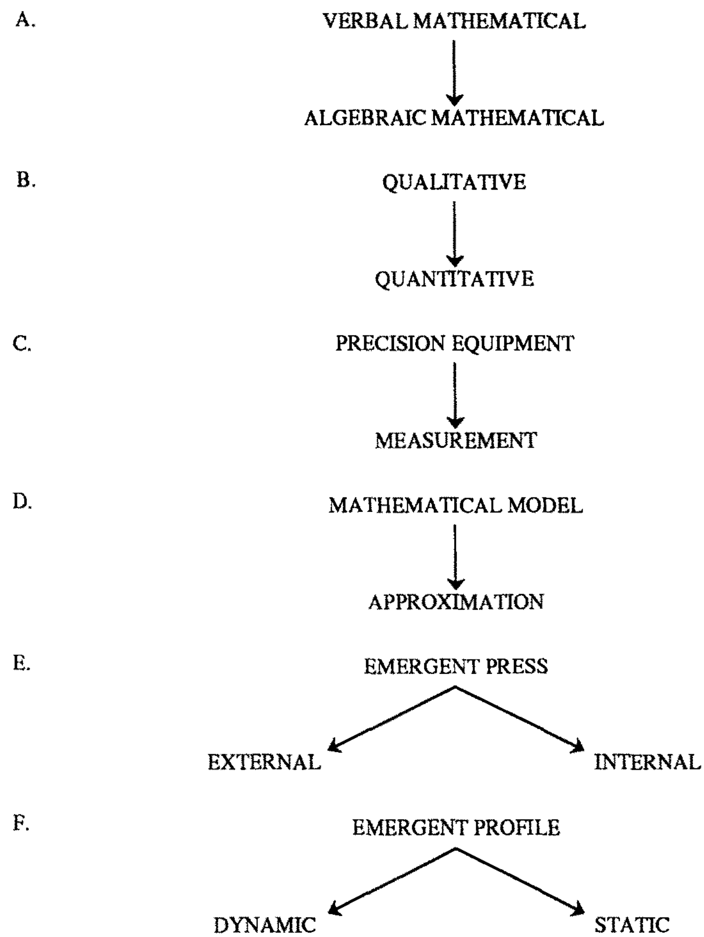


Fig. 1. Aspects of quantification extracted from the historical *P–V* profile.

historical statements are connected to the discovery of important scientific laws.

SUMMARY AND CONCLUSION

Student use and learning of the mathematical components of science courses has received criticism from science education researchers and philosophers because of the absolute dependence on algorithmic techniques in problem solving which does not generate understanding of the concepts involved. In addition to some of the approaches that have been adopted to correct this situation an approach based on a study of the historical profile of the mathematical component in question has been

Study the historical profile of the P-V relationship carefully and answer the following questions.

1. " Robert Boyle derived the mathematical relationship, $P \propto 1/V$, in 1660". Critically analyse this statement in the light of the historical profile presented and indicate whether you agree or disagree with this statement giving reasons for your answer.

2. Some educators consider it is important to understand quantitative relationships verbally before they are presented algebraically. Distinguish between a verbal description and an algebraic description and give an example of each. With reference to the profile, was the P-V relationship established verbally before it was established algebraically? Illustrate with examples from the profile. What is one advantage in expressing a quantitative relationship in algebraic form?

3. Using the date 1660 as a baseline (the date when Boyle reported the results of many qualitative experiments with air), complete the following table showing the number of years after 1660 for which the following events were reported according to the profile.

Event	Years after 1660
(a) Boyle's verbal statement of the reciprocal law.	________
(b) Use of the equation, PV=C, or its equivalent.	________
(c) Graphical representation of the law.	________
(d) Use of the equation, $V \propto 1/P$	________

4. You will notice from the profile that Thomson used the P-V relationship in the form, $dp/dv=-p/v$, in his thermodynamic study of the steam engine. Starting with the equation, PV=C, derive, using calculus, the derivative equation used by Thomson. What new information does this derivative equation reveal?

5. Edme Mariotte (a French physicist) is said to have discovered the P-V law independently of Boyle (English/Irish) some fourteen years after Boyle's discovery as shown in the profile. Discuss how this could have been possible in the 17th century. Would you expect such a situation to arise in today's scientific world? Comment.

6. Some people claim that developments in technology often lead to developments in our theoretical understanding of scientific laws. Is there any evidence of this from the profile? Comment.

Fig. 2. Example exercises arising from the *P–V* law historical profile.

discussed in this paper. This approach addresses the dynamic emergence of the scientific law and provides opportunities for increased student participation in the process.

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