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Twenty-Five Centuries of Quantum Physics: From Pythagoras to Us, and from Subjectivism to Realism

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Abstract. Three main theses are proposed. The first is that the idea of a quantum or minimal unit is not peculiar to quantum theory, since it already occurs in the classical theories of elasticity and electrolysis. Second, the peculiarities of the objects described by quantum theory are the following: their basic laws are probabilistic; some of their properties, such as position and energy, are blunt rather than sharp; two particles that were once together continue to be associated even after becoming spatially separated; and the vacuum has physical properties, so that it is a kind of matter. Third, the orthodox or Copenhagen interpretation of the theory is false, and may conveniently be replaced with a realist (though not classicist) interpretation. Heisenberg's inequality, Schrödinger's cat and Zeno's quantum paradox are discussed in the light of the two rival interpretations. It is also shown that the experiments that falsified Bell's inequality do not refute realism but the classicism inherent in hidden variables theories.

Key words: Causality, classicism, Copenhagen interpretation, hidden variable, phenomenalism, probability, quantization, quantum paradox, realism, subjectivism, superposition

1. Introduction

A quantum is a basic or indivisible unit, such as the cent in the American monetary system, the electric charge of the electron, and the bit of information. It is usually believed that quanta are peculiar to quantum physics, and that this was born only one century ago. I intend to disprove both theses.

In fact, the first to discover quanta was not Planck in 1900, but Pythagoras in the 6th century B.C. He did so while studying vibrating strings such as a harp's. Indeed, he found that the frequencies of such a string are integral multiples of a basic frequency or harmonic.

I also claim that a peculiarity of quantons – my name for the referents of the quantum theory – is not so much that some of their properties vary by jumps – which they do. It is that, save exceptions, their properties, such as position and energy, are spread out rather than sharp. More precisely, their values have probability distributions.

Another peculiarity of quantum physics is that it attributes physical properties to the electromagnetic vacuum. This is a fluctuating field with zero average intensities, that exerts a force on atomic electrons, causing their ‘spontaneous’ radiative decays to lower energy levels.

A third peculiarity of quantons is that, if they were once joined, they do not lose this association altogether: they do not become fully separable, or individually localized, however far away they may move from one another.

Quantons are certainly weird as judged by common sense. However, they share some properties with the referents of classical physics, or classons. One of them, certainly the most important, is that they exist independently of the observer’s mind. Hence quantum physics, contrary to a widespread opinion, does not call for a radical change in the realist theory of knowledge.

2. Classical Quantization

2.1. QUANTIZATION OF FREQUENCY: PYTHAGORAS TO D’ALEMBERT AND FOURIER

It is well known that the belief system of the Pythagorean brotherhood was a mixture of gold and dirt. One of its gold nuggets is the law that the possible frequencies of a vibrating string are integral multiples of a basic harmonic (frequency). That is, the possible frequencies of a vibrating string are υ, 2υ, 3υ, $\dots n\upsilon$.

The vibrating membranes and solids have similar properties. In all these cases the source of the discontinuity is the same: in a string (or membrane or cylinder) attached at the ends (or borders) there is room for only an integral number of stationary half- waves. In these and other cases, quantization is merely an effect of fixed boundary conditions. If these are relaxed, the waves triggered by an excitation, such as a blow, are progressive rather than stationary.

In sum, Pythagoras discovered the quantization of the oscillation frequencies of elastic bodies. This must be emphasized to debunk the myth that only exotic microphysical objects have quantal properties. Harps, drums, crystals, beams, bridges and many other large objects have some of them too.

The first to build a mathematical model of a vibrating body was Jean Le Rond d’Alembert (1747), the great mathematician and physicist who, together with Denis Diderot, edited the famous *Encyclopédie*, that challenged the established order. Two centuries later, the equation that bears his name is still one of the central formulas of theoretical physics. Thanks to d’Alembert we also know that, when a musician rubs a violin string, she makes it vibrate with an oscillation that equals the sum of vibrations of numerous frequencies and amplitudes: the violinist sets up a superposition of stationary waves.

Something similar happens of course with light waves. The strictly monochromatic waves are exceptional: in general, light waves are sums of waves of different amplitudes and frequencies. The case of white light is extreme: it is composed of light waves of all the frequencies capable of exciting the human retina. All these are examples of the superposition principle. This is actually a theorem in any linear wave theory, although it is often believed to be peculiar to the quantum theory.

If waves of all possible frequencies and amplitudes are added up, the resulting wave is a harmonic series, invented in 1822 by Joseph Fourier – no relation to Charles, the utopian socialist. In fact, almost any function or curve, oscillation or wave, whether stationary or progressive, may be analyzed as a Fourier series (or integral). Every one of the terms of this series, such as $\sin n2\pi \upsilon_0 t$, represents an elementary wave (or oscillation), which is an integral multiple of the basic frequency υ_0. Thus, paradoxically, continuity results from an accumulation of discontinuities – a case of emergence.

Fourier's work culminated then a process of discoveries and inventions began by Pythagoras, and reinitiated by d'Alembert more than twenty-two centuries later. These are examples of what may be called Merton's law (1968): Every discovery or invention has some precursor. In turn, this law exemplifies Lucretius's: Nothing comes out of nothing.

2.2. QUANTIZATION OF ELECTRIC CHARGE: FARADAY TO MILLIKAN

In his experimental study of electrolysis, Michael Faraday discovered in 1833 that the chemical effect of an electrolytic current – that is, the quantity of matter deposited on an electrode – is proportional to the quantity of electricity involved. In turn, in the light of Dalton's atomic theory, that quantity is seen to be an integral multiple of a certain basic or elementary charge. That is, the electric charge is quantized. In 1911 Millikan found that the unit of electric charge is the charge of the electron, which had been discovered in 1889. To put it in the negative: there are no bodies with a fractionary electric charge.

We are so used to this result that we do not stop to think that is just as surprising as would be the finding that there is also a natural unit of mass, so that the mass of any particle or body would be an integral multiple of the mass of some elementary particle. It is no less amazing that the quantum theory does not contain an operator representing the charge quantization. This seems to be a gap to be filled: if we were good Pythagoreans we would craft a quantum theory of the electrostatic field, in which the electron charge would appear as the quantum of electricity.

3. Modern Quantization

3.1. ENERGY QUANTIZATION: PLANCK, EINSTEIN, AND BOHR

In 1900 Max Planck postulated, somewhat reluctantly, that a black body, such as a microwave oven, does not absorb or emit radiant energy in arbitrary quantities but in lumps. More precisely, the quantity of electromagnetic energy of frequency υ is an integral multiple of the basic quantity of energy $h\upsilon$, where $h = 6.626 \times 10^{-27}$ erg.sec, the famous Planck constant.

A peculiarity of this constant is its extreme smallness compared to the actions characterizing everyday processes. (Recall that 1 erg.sec is the action spent in pulling a one-gram marble over the distance of 1 cm at the speed of 1 cm per sec.)

Another peculiarity of h is that it is universal, that is, its value does not depend on the kind of matter. (Other such constants are G, c, e, and k.)

Five years later, Albert Einstein postulated that something similar holds for radiation in free space: that the total energy of a light beam of frequency υ is $nh\upsilon$, where n is a positive integer. In other words, radiation is composed of photons, or electromagnetic field quanta (This only holds for radiation: it does not hold for the electrostatic or magnetostatic fields.)

Moreover, the discovery of the Compton effect in1923 confirmed Einstein's further hypothesis, that the photon has a momentum (namely $h\upsilon/c$), so that it resembles a particle. However, a beam of visible light of one erg is constituted by about one trillion photons. No wonder that it can be described to a good approximation by Maxwell's classical equations. Only far weaker light beams call for quantum electrodynamics.

In 1911 Ernest Rutherford explained the result of his scattering experiments by assuming that an atom is made up of a positively charged hard core surrounded by electrons. Niels Bohr (1913) mathematized Rutherford's model and joined it with Planck's and Einstein's ideas about radiation. To acomplish this feat, he added the heterodox postulate that the states of a stable atom are denumerable. Every one of these states is characterized by a positive integer, and it corresponds to the trajectory of an electron orbiting around the nucleus.

In technical jargon, Bohr postulated that in an atom the action (energy $\times$ time) is quantized and, more precisely, that it is an integral multiple of the Planck constant h. This implies that a transition between two adjoining stable states is discontinuous. Such event is a quantum jump, in which the atom gains or loses the quantity of energy $h\upsilon$, according as it absorbs or emits a photon of the same energy. The expression 'quantum jump' has of course been with us ever since. However, let us not forget the injunction to try and analyze every such jump as a continuous albeit swift process. This may hold, in particular, for the so-called collapse of the state function caused by a measurement.

The Rutherford–Bohr planetary model of the atom proved initially to be so successful, and it became so popular, that it is still the logo of modern physics. This, despite having become obsolete three quarters of a century ago. Indeed, Bohr's theory is only half-way quantal, because it retains the classical ideas of orbit, shape, size, and sharp energy value. These characteristics become blurred in quantum mechanics, although they reappear gradually in the case of heavy atoms. In other words, the geometric properties of matter are not fundamental, but emerge as the system gets more complicated.

3.2. CLASSICAL AND QUANTAL VARIABLES

Louis de Broglie, Werner Heisenberg, Max Born, Pasqual Jordan, Erwin Schrödinger, Paul A.M. Dirac and a few others built the modern quantum between 1924 and 1930. (I met three of the founders – an indicator of either the recency of the theory or my advanced age.) This theory kept the classical concepts of space, time, mass, and electric charge. On the other hand, it gave up the classical concepts of position, linear and angular momentum, and energy. Instead of these, it introduced operators that act on the famous state function ψ, formally similar to a classical wave, which is why it is also called wave function.

This formal resemblance suggested, at the beginning, that matter is wave-like: there was talk of matter waves. In 1927 Davisson and Germer confirmed experimentally this conjecture under certain conditions. However, under different conditions the corpuscular aspect stands out. One therefore talks about the particle-wave duality. This duality is obvious in de Broglie's equation $p = h/\lambda$. It is no less evident in the electron microscope (1933), where electrons are shot like bullets but end up diffracted like waves.

The referents of quantum mechanics are therefore neither particles nor waves. They are something *sui generis* that deserve a name of their own. I have proposed to call them *quantons*.

The wave-particle duality pops up clearly in Heisenberg's inequality, popularly misnamed indeterminacy or even uncertainty relation. According to it, the position and the linear momentum have distributions whose variances (or mean standard deviations) are inversely proportional to one another. More precisely, $\Delta x \cdot \Delta p \geq h/4\pi$. That is, the sharper the position (small Δx), the more spread out the momentum (large Δp). If a quanton is well localized, it lacks a sharp velocity value; and if it has a sharp velocity value, it is not well localized. (Note that I am tacitly regarding these scatters as objective properties of quantons, not as measurement errors that might conceivably be reduced with the help of better equipment.)

The quantum angular momentum $x \times p$ is even stranger: if one of its components has a precise value, then the other two are totally blurred. Hence, the angular momentum is not a vector (or rather tensor) proper. Nor are the spin and the velocity in relativistic quantum mechanics. The quantal arrows are so blurred in both breadth and direction, that they do not look at all like arrows.

4. Strange Matter

4.1. CLASSONS AND QUANTONS

The preceding discussion suggests the following classing of the types of matter:

> *Classons* (e.g., intense light beam, DNA molecule, cell, rock, planet)
>
> *Quantons* (e.g., photon, electron, atom, black body, superconductor)

Actually this is not a partition, because there are things intermediate between classons and quantons, such as weak light beams and medium-size molecules, for instance, Carbon 60. They are often called mesoscopic objects; we may also all them semiclassons or semiquantons. Unsurprisingly, such things are described by semiclassical theories. (Actually even the standard quantum-mechanical theory of atoms is semiclassical, in that it leaves the electromagnetic field unquantized.)

A feature of semiclassical theories is that, unlike the quantum- theoretic ones, they allow for pictures of sorts. For example, the trajectory of the external electron of an atom in a highly (or Rydberg) excited state can be pictured in two different ways: as a microplanetary orbit, or as a stationary circular orbit with a number of crests equal to the principal quantum number.

Besides semiclassons (or semiquantons), there are concrete or material things, such as organisms, robots and social systems, that are beyond the reach of the quantum theory – *pace* the radical reductionists who believe that this is a universal theory. Those things escape the quantum theory not because they are large, but because they have supraphysical properties, such as those of being alive, behaving by proxy, and obeying norms that do not derive from physical laws. We should be glad to have such a general and accurate theory as the quantum theory, but it would be silly to try and apply it beyond its domain.

4.2. SUPERPOSITION AND MEASUREMENT

The superposition 'principle' is the theorem acording to which, if two or more functions are solutions of a given (linear) differential equation, then their linear combination too solves the same equation. In physical terms: the superposition of simple (in particular stationary) states is a state. This theorem raises some perplexities. Let us consider one of them, namely whether the principle is consistent with the conservation of energy.

Suppose a certain isolated quanton is not in a stationary state with a sharp energy value, but instead in a state constituted by several sharp energy states, each with a given weight (or probability). To simplify, assume that only two stationary states, to be called ψ_1 and ψ_2, contribute to the total state. That is, assume that the quanton has two possible energies, E_1 with probability p_1, and E_2 with probability p_2. (Obviously, these probabilities add up to unity.) In other words, the energy distribution has two peaks, one at E_1 with height p_1, and the other at E_2 with height p_2. (That is, the state space has only two axes, and the state function is a vector ψ with two components, ψ_1 and $\psi_2 : \psi = a_1\psi_1 + a_2\psi_2$, where $|a_1|^2 = p_1$ and $|a_2|^2 = p_2$.)

According to John von Neumann (1932), if an energy measurement is made on the quanton, the original superposition will be projected onto the axis 1 of the state space with probability p_1, or onto the axis 2 with probability p_2. In the first case the experimenter will find the exact value E_1, and in the second case the exact value E_2. (If a large number N of measurements are performed on quantons in the same state, roughly p_1N of them will yield the energy value E_1, and p_2N the value E_2.) To sum up, before the measurement is performed, the energy of the quanton had two values, each with its own probability (or weight or tendency), and the measurement act 'selected' one of them and eliminated the other.

Was the quanton energy conserved? Certainly, the theory includes the theorem that the energy of an isolated quanton is a constant of the motion. But our case does not satisfy the tacit condition of the theorem, namely that the energy has a single sharp value. And, obviously, the energy cannot be conserved if it does not have a definite value to begin with. Besides, the measurement in question interferes strongly with the quanton to the point of reducing its state function, which violates the condition that the quanton remains isolated. It is, indeed, a demolition experiment.

This example shows that, to perform an energy measurement, and in particular to test the corresponding conservation theorem, the quanton has got to be suitably prepared. More precisely, it must be put in a state characterized by a sharp energy value, such as either E_1 or E_2. Only thus will a measurement ascertain whether the energy has remained constant. But this measurement will have to be non-intrusive, like the ones performed in spectroscopy. That is, the only measurements considered by von Neumann are of the demolition kind: they involve the sudden and non-causal reduction of the wave function, and therefore do not serve to test the conservation theorems (or constants of the motion).

The alternative would be to sacrifice the conservation theorems on von Neumann's hallowed altar. But such sacrifice would not even please the so-called Copenhagen ghost. Indeed, the conservation laws are entailed by basic law statements. If those were to fail, these too would fail, and the universe would be chaotic in the ordinary sense of the word. (Indeed, from 'If B, then C' and 'not-C' it follows that 'not-B'.)

5. Orthodoxy and Heterodoxy

5.1. THE ORTHODOX OR COPENHAGEN INTERPRETATION

For a while the fathers of quantum mechanics calculated state functions ψ without knowing what these represented. That is, they had mastered the syntax of the theory but ignored its semantics. It was only in 1927 that Max Born proposed the interpretation that bears his name and is currently accepted. (This was the first time that the Nobel prize was awarded for a contribution to semantics.)

The interpretation in question reads thus: the quantity $|\psi(x, t)|^2$ is the probability of *finding* the quanton inside the unitary volume placed at point x when its position is measured at time t. This postulate shows, among other things, that the probability concept is basic in quantum mechanics: that is, it is not deduced from non-probabilistic assumptions. It also suggests that the probability in question depends upon the observer as much as upon the object observed.

What happens when no position measurement is being performed? What is the meaning of $|\psi(x, t)|^2$ then? According to the standard (Copenhagen) interpretation, in this case the quanton *has* no position, not even inside the volume element being considered. The idea is that you won't find unless you search, and that what is not found does not exist. In general, it is said that a quanton that is not being measured lacks properties: that it acquires them only when they are measured, which in turn depends on the experimenter's decision. (Oddly, this holds only for the so-called observables, i.e., dynamical variables, such as x and p: it does not hold for mass and charge.)

The Heisenberg inequality, which we first met in Section 3, used to be interpreted as follows. It was said that the mean standard deviations or scatters Δx and Δp are the effects of the measurements of x and p respectively. For example, to localize an atom we shed light on it, which event causes the displacement of the atom as a consequence of receiving a kick. This is what we still read in most textbooks.

Notice that this interpretation presupposes that the quanton has a sharp position and a sharp momentum before the measurement, only we do not know them. It also presupposes that causality rules on the quantum level. However, neither presupposition agrees with the philosophy reigning in the physics community at the time the quantum theory was born. This philosophy, operationism, was clearly formulated by Percy W. Bridgman in his 1927 best-seller, *The Logic of Modern Physics*. From that year on until 1938, the same philosophy was expanded by the members of the Ernst Mach Verein, later known as the Vienna Circle, the cradle of logical positivism.

To get around that objection, in 1935 Niels Bohr and Werner Heisenberg, with the support of Max Born and Wolfgang Pauli, and the blessing of the Vienna Circle, proposed the so-called Copenhagen interpretation. According to it, the measurement of a variable does not disturb its pre-existing value: it creates it. Put negatively, the quanton has no dynamical properties as long as it is not being measured. (But, again, it may possess mass and charge.)

Hence the thing does not exist except as a component of a sealed and unanalyzable unit: object (quanton)-apparatus-subject (experimenter). As Leon Rosenfeld – Bohr's closest coworker – once put it, the experimenter 'conjures up' at will the quantum-mechanical object at a given position or with a given velocity. Were it not for the physicists there would be no atoms, not even in their own eyes. This would hold for all physical objects. For example, the Moon would not exist as long as no one is gazing at it.

In general, the experimenter would create the world as she measures it. To be is to measure or to be measured. This would be the new version of the maxim that George Berkeley had crafted in 1710: "To be is to perceive or to be perceived". No wonder that half a century later the post-Mertonian sociologists of science, such as Bruno Latour, Steven Woolgar, Karen Knorr-Cetina, Harry Collins, and other contributors to the quarterly *Social Studies of Science*, have felt justified in claiming that scientific facts are constructed by scientists or scientific communities.

Clearly, this view is anthropomorphic and even magical. It collides head-on with the realism inherent in both common sense and the practice of science. In particular, it is inconsistent with the tacit assumption of scientific research, that nature satisfies objective laws that precede scientists, who only attempt to discover them while minimizing their own impact on the things studied.

What are the sources of the anthropomorphic component of the Copenhagen interpretation? I suggest it has two roots. One is the fact that the microphysical quantal events are imperceptible without the help of amplifiers. Surely they happen everywhere all the time, as shown, e.g., by the success of astrophysics. But they can only be detected or produced in a suitably equipped and manned laboratory. However, from the fact that an experimenter can 'conjure up' quantum effects, it does not follow that these only happen under experimental conditions. A first source of the subjectivism inherent in the Copenhagen interpretation is thus a mere logical fallacy.

Another source of the orthodox interpretation of the most unorthodox of theories is, as mentioned earlier, the positivist philosophy that reigned at the time the theory was being built. According to that philosophy, which grew out from Ernst Mach's, things only exist which can be measured; while actually measurability is only a sufficient condition, hence an indicator or criterion, of real existence. Thus, the second source too turns out to be a logical fallacy. We shall return to this theme towards the end. Let us now pursue our examination of the differences between quantum and classical physics.

5.2. THE BOHR–EINSTEIN CONTROVERSY: WHO WAS RIGHT ABOUT WHAT?

In 1935 Einstein and Bohr held a memorable debate in *Physical Review* on the interpretation of quantum mechanics. They resumed it in 1949, in the volume edited by P.A. Schilpp devoted to Einstein. Both discussants tackled in particular the questions whether physical theories should represent reality such as it exists independently of the inquirer (Einstein, yes, Bohr, no); whether the quantum theory is essentially complete (Einstein, no, Bohr, yes); and whether the theory should be completed with the addition of 'hidden' (that is, scatter-less) variables (Einstein, yes, Bohr, no).

The prevalent opinion is that Bohr won the great match: that the state function contains all the necessary information, and yet it does not represent reality but rather appearances to the experimenter. Only a handful of heretics, headed in 1951 by David Bohm and Louis de Broglie, and joined later on by John S. Bell and a few others, thought that Bohr was wrong, and set out to complete the theory the way Einstein had suggested. In particular, Bohm enriched standard non-relativistic quantum mechanics with a classical position coordinate and the corresponding momentum, as well as with a *sui generis* potential.

Note that the variable x occurring in standard quantum mechanics is not a time-dependent function representing a property of the quanton. It is, instead, the same 'public' geometric coordinate that occurs in field theories: it identifies a generic point in space. (Hence, contrary to what happened in Heisenberg's matrix mechanics, the variable x that occurs in the standard theory, centered in Schrödinger's equation, it is not an operator, and therefore it has no eigen functions. True, one can compute its rate of change, but only through the hamiltonian and the state function. Bohm's theory contains both position coordinates, the geometric and the classical dynamical or time- dependent position coordinate. And, as mentioned earlier, it also contains a potential whose gradient is a strange internal force absent from both standard quantum mechanics and classical physics. As we shall see in Section 8.1, Bohm's attempt eventually met with defeat in the laboratory. However, let us now proceed with the famous debate.

In my view, each of the giants lost three points and won one:

(a) Bohr was right in stating that quantum mechanics holds, at least to a very good approximation, without the addition of hidden (classical) variables; but he was wrong in holding that quantum mechanics fails to describe a reality independent of the inquirer.
(b) Einstein was right in demanding that every physical theory should represent reality as truly as possible; but he was wrong in suggesting that it was necessary to 'classicize' quantum mechanics, and in particular to enrich it with precise trajectories.
(c) Neither Bohr nor Einstein were right with regard to completeness, since no factual (empirical) theory, however exact, may cover its referents in all detail. It is likely that there will always some holes left, hence room for improvement.
(d) Neither Bohr nor Einstein characterized clearly the key philosophical concepts of reality and causality, that occurred at the very core of their debates.

Besides, in their 1949 exchange, Bohr misled Einstein into persuading him, with the help of a gedankenexperiment, that there is a Heisenberg inequality for energy and time, namely '$\Delta E.\ \Delta t \geq h/4\pi$'. But the axioms of quantum mechanics entail no such formula, and for a simple reason: in this theory, time is a classical (or 'hidden') variable, that is, $\Delta t = 0$ for every possible state function of an arbitrary quanton. Besides, no moderately complicated theoretical formula can be inferred from an analysis of experiments, not even real ones – especially if it does not contain empirical parameters. In particular, the Heisenberg inequality and its kin derive from the postulates of quantum mechanics, which are so general that they make no reference to any measurements.

In short, neither of the two giants won. Yet, they succeeded in stimulating the debate over the foundations of quantum mechanics – and in muddling the philosophical issues.

6. Determinism and Indeterminism, Atomism and Plenism

6.1. CAUSALITY AND PROBABILITY

In classical physics, chance only emerges in large aggregates of things or events of some kind, that behave individually in a causal way but fairly independently from one another. Trivial examples: molecules in a low-density gas, suicides in a nation, and automobile accidents in a city. By contrast, in quantum physics chance emerges not only at the crossing of independent causal histories, but also at the individual level – so much so, that the basic state functions refer to individuals, not statistical aggregates. For example, every atom in an excited state has a certain probability of decaying to a lower energy state within the next minute; and every electron in an electron beam has a certain probability of being scattered by a certain target within a given solid angle. No predestination here.

In other words, the state function is basic, not derived. This holds even in theories which, like Bohm's, contain scatter-less dynamical variables. This fact is usually regarded as a triumph of indeterminism. However, this interpretation is wrong, because indeterminism proper denies the existence of laws and affirms instead that anything can happen. By contrast, quantum mechanics is centered in laws and excludes a number of conceptually possible things and events, such as the formation of particles out of nothing and the reabsorption of a photon by the atom that emitted it.

Note further that some quantum-theoretical laws are not probabilistic. Examples: the principles of conservation of energy and angular momentum; the so-called laws that 'forbid' certain transitions between atomic levels; and the exclusion principle, that denies the possibility that two electrons (or other fermions) in a system occupy exactly the same state.

Moreover, the concepts of chance and causality occur together in such sentences as 'the probability that cause C will produce effect E equals p', which litter the theories of scattering and radiation. Besides, the random fluctuations of the electromagnetic vacuum (no photons present) cause the 'spontaneous' emission of light by atomic electrons in excited states (the Lamb effect).

In short, causation and chance intertwine in quantum mechanics. This intertwining is clear from the state equation, where the term $H\psi$ occurs. Indeed, the energy operator H is the causal factor, since it contains the potential (or source of the forces or efficient causes), whereas ψ represents the chance factor, by virtue of Born's principle.

For these reasons, it is more correct to talk of the broadening of determinism than of its demise – as I argued in my book *Causality* (1959). In this broad sense, determinism may be defined as lawfulness together with Lucretius's principle, *Ex nihilo nihil fit*.

6.2. PLENISM AND ATOMISM: WHICH ONE TRIUMPHED?

Another popular myth is the belief that atomism triumphed over the plenism of Aristotle and Descartes. No such thing happened. First, because all fields are continuous media: they are extended substances, not aggregates of particles. In particular, the quanta of the electromagnetic field – the photons – are not point-like corpuscles but extended pieces of matter without sharp boundaries. Only their energy has been quantized, but energy is a property, not a thing. Second, the atomic nuclei, atoms, molecules and solid bodies only exist by virtue of the fields that hold their constituents together.

Third, the basic quantum theory is not quantum mechanics but the so-called second quantification, a field theory. In this theory, the electrons and other elementary particles are conceived of as the quanta of the respective field (e.g., electronic and electromagnetic). Moreover, as already mentioned, quantum electrodynamics postulates the existence of a residual electromagnetic field, of null average intensity but capable of causing a number of well-attested effects, among them the 'spontaneous' radiative decay of atoms.

In short, certainly there are corpuscles, but they have a wave-like aspect. Besides, they are quanta of fields. The resulting view resembles somewhat Descartes's, which was also a synthesis of Aristotelian plenism and Democritean atomism. But of course the quantum-theoretical synthesis, unlike the Cartesian, is computable and experimentally confirmed. In fact, it is the most accurate scientific theory ever constructed.

7. Two Paradoxes

7.1. SCHRÖDINGER'S CAT: STILL HALF-ALIVE AND HALF-DEAD?

In 1935 Erwin Schrödinger, one of the founding fathers of wave mechanics, designed a gedankenexperiment designed to cast doubt on the soundness of his own brain-child. This was the famous Schrödinger's cat, which continues to elicit passionate debates, as well as feeding an army of philosophers. Let us take a look at it.

Suppose a live cat is locked in an iron cage together with a small sample of radioactive material and a phial containing a powerful poison. The disintegration of a single atom suffices to break the vial and thus kill the cat nearly instantly. Thus, the unfortunate cat's life depends on pure chance. Now, according to the Copenhagen doctrine, nothing will happen as long as the cat continues to be locked up, since in this case it won't be observed. During this lapse the cat is deemed to be literally half-alive and half-dead. Or, if preferred, it will oscillate between life and death.

In other words, while in the cage the cat's state function will be a superposition (linear combination) of the alive and dead states (remember Section 4). It is only when the observer opens the lid and looks inside, that this sum will collapse onto either of its components. It is not that we have to wait until the cage is opened to find out what has become of the cat – as a naive biologist would think. No, it is only then that the cat will either resume its full life or die for good. This fable is often recalled as an example of both the principle of superposition, and the collapse or reduction of the wave function upon measurement.

Schrödinger, a cat lover as well as a great physicist, smelled here a rat. He thought the theory was wrong. In my opinion what is wrong is the positivist (or subjectivist) interpretation of the theory. A realist might object as follows. First, no one has a clue as to how to describe a cat in quantum-mechanical terms – or even a much simpler system such as any of the proteins in the cat's whiskers. Hence, writing a linear combination of the states for a living cat and a dead cat amounts to uttering the sentence 'Blah plus bleh equals blih'. Quantum mechanics does not hold for living beings, and not because these are macrophysical, but because they have properties that the theory ignores. In particular, the theory cannot explain why cats metabolize and reproduce, let alone why they like to hunt and purr.

Second, there is a way of finding out what happens inside the cage without opening the lid or interfering with the process, namely, to have a camera film the entire process. Only a follower of Berkeley's would believe that the cat's fate depends on whether the camera is working or on the opening of the lid. The cat will live or die according as at least one of the radioactive atoms will decay. And since such decay happens in its nucleus, which is well protected by the electronic armor surrounding it, the event takes place regardless of what the observer may do.

(It may be objected that in 1996 Chris Monroe, David Wineland and coworkers, succeeded in carrying out Schrödinger's experiment on a single beryllium ion trapped in an electromagnetic cage. Indeed, they excited the ion into a superposition of two spatially separated quantum states. However, the system in question was not macrophysical but mesophysical, and neither of the states in question had anything remotely resembling life.)

In sum, the Schrödinger cat paradox only shows that the quantum theory is not a theory of everything, and that its Copenhagen interpretation is absurd. It does not occur in a realist interpretation.

7.2. IS ZENO BACK?

Twenty five centuries ago, Zeno of Elea thought he had proved the impossibility of motion. He noted that, to go a given distance, one must first cover one half, then one half of the remaining half, and so on successively and indefinitely. And he thought that the sum of these infinitely many though dwindling paces had to be infinite, and therefore physically and mathematically impossible. It took two millennia to find out that the infinite series in question converges to a finite value, namely the given distance.

The Copenhagen interpretation lends itself to a similar paradox, which has a precedent in the English saying 'The watched kettle never boils'. Anyone knows that this refers to our impatience. But a Copenhagen fanatic is bound to take that saying seriously, since it concerns the possibility of an observer's 'conjuring up' a physical process. He will claim that, as long as the kettle is not being observed, it is in a state that is the linear combination of the 'boiling' and 'not boiling' states, similar to the case of the mythical cat. And he will add that this sum will reduce to one of its two terms the moment the cook glances at the kettle. He will also say that something similar happens with any other unstable system, such as a radioactive nucleus or an atom in an excited state.

Curiously, similar arguments have recently been used in favor of the so-called anti-Zeno paradox. That is, the kettle would boil earlier if it were observed. But so far neither of the two effects has been observed. Nor are further observations really necessary, and this for the following reasons.

First, the belief in the causal efficacy of the gaze evokes the ancient Greek view of vision as the emission of light by the eye, refuted a millennium ago by Alhazen.

Second, the boiling of the kettle, as well as the radioactive and radiative decays, are not instantaneous events, but outcomes of complex events that, however swift, take some time. (Such processes are supposed to be described by the time-dependent Schrödinger equation, which does not involve quantum jumps, rather than by the Schrödinger equation for stationary states.)

Third, the 'observations' invoked in support of both the Zeno and the anti-Zeno quantum effects are not observations: they are experiments that disturb the state of the atoms concerned.

Fourth, no observer occurs in the theories describing these processes. The observer is a parasite foisted upon the quantum theory by the positivist philosophy of Berkeley, Mach, Bridgman, and the Vienna Circle. According to it, everything that happens in the world is the doing of some observer.

In short, the quantum theory has not resuscitated Zeno of Elea. The world keeps on going despite the subjectivist philosophers and the physicists seduced by their fallacies.

8. Hidden Variables

8.1. BOHM AND BELL

As recalled in Section 5.2, in 1952, at the suggestion of Einstein, David Bohm enriched quantum mechanics with two 'hidden' variables, that is, scatter-free ones: a classical position coordinate $x(t)$ and the corresponding linear momentum $p(t)$. He thus produced a new theory, although he was under the wrong impression that he had only reinterpreted the standard theory. Two formulas of the new theory stood out: those for the trajectory of a quanton, and for the force that acted on it even in the absence of external forces. Being exotic and unmeasurable, this quantum-theoretic force drew the curiosity of parapsychologists and oriental mystics.

Bohm also thought that his theory was causal, while in fact it was, just like the standard theory, semicausal and semiprobabilistic. Indeed, it kept the state function

ψ instead of defining it in terms of 'hidden' variables. On the other hand, Bohm's theory was indeed realistic: no observer-dependent variables or events occurred in it.

This theory made quite a stir: indignation among the Copenhagen faithful, and enthusiasm in the realist camp, which included Einstein and de Broglie among others. After Bohm answered to my satisfaction the thirty or so objections I raised against his theory, I adopted it and taught it for a while. My students hailed it enthusiastically because it seemed to explain in a causal fashion many of the processes that the standard theory treats as black boxes. In particular, the theory seemed to explain electron diffraction in terms of Bohm's queer quantal internal force, that changes quickly from place to place, causing fast fluctuations on the quanton's motion.

The orthodox school were not amused. In particular, the redoubtable Wolfgang Pauli objected that the new theory failed to account for measurement, which was thought to be accounted for by von Neumann's projection postulate. (Recall Section 4.2.) But neither they nor Bohm realized the impossibility of a general theory of measurement. Indeed, since there are no universal meters, there can be no universal theory of measurement either.

Every measurement device calls for its own special theory, and such theory must reveal the mechanism at play – such as ionization in the Wilson chamber, and a photochemical reaction in that of the photographic plate. Moreover, every such theory must be a fusion of fragments borrowed from quantum and classical theories, since it must bridge imperceptible microphysical events and detectable macrophysical ones – as Bohr had rightly emphasized.

To be sure, every time an exact measurement is performed, the wave function of the measured object must be reduced or projected, as postulated by von Neumann. Otherwise no sharp values (up to the random experimental error) would ever be measured. However, such reduction is unlikely to be instantaneous. And, above all, its mechanism cannot be the same for all kinds of measurement.

In fact, the various kinds of measurement should be described by different theoretical models, every one of which should focus on a distinct reduction process caused by the interaction between the apparatus and the object under measurement. For example, measuring wavelengths with a comparator is not the same as measuring the intensity of a radioactive source with a Geiger counter. (Even Pauli, a Copenhagen apostle, had admitted that there are two types of measurement: obtrusive and non- obtrusive.)

Bohm was joined by a few other physicists eager to restore both realism and causality, among them John S. Bell. However, interest in the new program declined because there were no new experimental results relevant to hidden variables theories. This was soon to change, as will be seen anon.

8.2. DEMISE OF HIDDEN VARIABLES: ASPECT ET ALII

In 1966, John S. Bell proved one of the inequalities that bear his name, and that set an upper bound on certain probabilities if local hidden variable theories are true. These formulas are justly famous for two reasons. First because, unlike all the other physical formulas, they hold for an entire family of theories rather than for a single one. Second, because they enabled the design of crucial experiments to decide between the standard quantum theory and the family of local hidden variables theories.

In 1972, Stuart Freedman and John Clauser performed the first experiment to test one of the Bell inequalities. And in 1981 Alain Aspect triggered an avalanche of highly publicized experiments of the same kind. All of them gave negative results: they falsified Bell's inequalities and, with them, the whole family of local hidden variables theories.

However, the debate did not stop there. Indeed, Aspect, possibly influenced by both Einstein's conflation of realism with classicism, as well as by Bernard d'Espagnat's phenomenalism, interpreted his own results as falsifying realism. Even *Science* announced the demise of realism. Actually, as already hinted, the alleged realism was nothing but the classicism defended by Einstein. One feature of classicism, inherent in the famous 1935 paper by Einstein, Podolsky and Rosen, or EPR for short, is the demand of the predictability of every individual event. Another is locality, as defined by Einstein in 1949.

Obviously, the experimental falsification of the Bell inequalities confirmed standard quantum mechanics, which predicts the converse inequalities. In particular, they confirmed the quantum- theoretical hypothesis of the reality of chance, as is obvious in Born's formula about the meaning of the state function, as well as in any formula about the probability of a transition between two atomic levels. However, the objective chance hypothesis makes no dent on realism: it only broadens it. Already Epicurus, a realist as well as a materialist, had assumed that atoms deviate spontaneously from the straight line.

As for locality, it means that, at least in principle, every thing can be isolated from the rest, and consequently every event can be confined within a region of space. The whole of classical physics complies with this requirement. A consequence of locality is separability: two things that are initially united can become separated until each of them behaves independently of the other. The reason is that all classical interactions weaken rapidly with increasing distance. (Not all forces behave this way, though. For example, the elastic force in an oscillator, as well as certain forces postulated in particle physics, increase with distance.)

By contrast, as Einstein and Schrödinger first noted with dismay, quantum physics is not local. Consequently, the components of a system are never fully separable: *Once a system always a system*. For example, if a system is divided into two parts that become mutually separated, what is done to one of them is bound to affect the other, almost as if they were still joined. (Human analog: the Briton who travels alone to Australia for a lecture remains faithful to his or her spouse, to whom he or she is attached.)

This is a counter-intuitive fact, that cannot be explained in terms of unknown action at a distance forces. We had better come to terms with it, the same as we have come to accept energy quantization, tunneling, and the wave aspect of 'particles'. It is part of the quantum-theoretical package, just as unsettling to ordinary knowledge as it is exciting to the scientific imagination. On second thought, it is not more baffling than boat flotation, jet propulsion, the birth of photons, or electromagnetic levitation.

9. Phenomenalism and Realism

9.1. APPEARANCE AND REALITY

Realists hold that (a) the world external to the inquirer exists independently (ontological thesis); and (b) reality can and must be objectively described (epistemological thesis) – unless of course one is trying to describe a human's subjective experiences, which is fine with psychologists and fiction writers but not with physicists.

By contrast, the phenomenalists claim that there are only phenomena, i.e., appearances to some subject, or at least that only they can be known. Consequently, they hold that the function of natural scientists is to account for appearances instead of exploring the world as it is, independently of the inquiring subject.

For example, since we see the Sun turning around our planet, and not the other way round, planetary astronomy should be geocentric or Ptolemaic, not heliocentric or Copernican. And, since in a particle accelerator only what is shot at the target and what comes out of it are detected, one should abstain from speculating about the forces at play during the collision process. In general, one should favor black boxes over translucent ones, and thus abstain from asking the most interesting questions.

Some of the greatest realist heroes are Democritus, Aristotle, Galileo, Boltzmann, Planck, and Einstein. The phenomenalist heroes are Ptolemy, Hume, Kant, Mach, Duhem, and Bohr. Which of the two parties does quantum physics vote for? If we consult the philosophical writings of Niels Bohr, Max Born, Bernard d'Espagnat, the young Werner Heisenberg, Wolfgang Pauli, Eugene Wigner, and other famous physicists, it turns out that the phenomenalists outvote the realists. But if we analyze the formulas that those same physicists handle, it turns out that the realists win.

Indeed, those formulas contain concepts denoting imperceptible things and properties, such as those of field, electron, neutrino, atomic number, mass density, electric charge, state (in particular ground state), probability, scattering cross section, dissociation energy, and valence. By contrast, the quantum theory does not deal with appearances or phenomena, such as tastes, smells, colors, or optical illusions – all of which are colored by learning, expectation, and feeling.

Appearances happen in the brain, not in the physical world. This is why they are studied by neuroscience and psychology, not physics. For example, physicists know about light frequencies, not about color sensations; and chemists know about molecules, not about the smells we feel when inhaling them. As Galileo and Locke put it, physics studies primary properties, not secondary ones. The relations between primary and secondary properties are studied by psychophysics. And this science is a merger of two disciplines, not the reduction of one of them to its partner.

Regrettably, most philosophers and sociologists of science do not study the formulas or the experiments themselves, but only the amateur philosophical comments of physicists. So, they are easily misled. In particular, they tend to believe the philosophical 'conclusions' that scientists claim to extract from their own work, while in fact they learned them from philosophers.

In conclusion, physics is not phenomenalist but realist. However, it is not everywhere strictly causal or local.

9.2. QUANTUM REALISM

I argued above that the standard or Copenhagen interpretation of the quantum theory is anthropomorphic, whereas scientific practice is realist. Indeed, scientists explore the world and attempt to keep at arm's length from the things they handle or model, because they are intent on discovering what they are like in themselves rather than for ourselves. Only technologists treat everything in the world in relation to man's needs and desires, and thus design bridges between man and his natural and social worlds. In short, unlike science, technology is anthropocentric. However, neither is phenomenalist, because appearances are circumstantial, shallow, and often misleading.

For example, instead of interpreting Born's postulate in terms of the probability of *finding* the quanton in question within the volume element Δv, the realist will say, along with de Broglie, that the probability in question is the likelihood of the quanton's *presence* in the given region. Moreover, he will distinguish the two probabilities of presence, when no position measurement is being performed, and when it is performed. These two numbers are bound to be different because in one case the quanton is subject to a perturbation that depends on the measurement technique, whereas in the other it is not. In other words, there are two different probabilities at stake: the calculated and the measured. And the point of the measurement is to contrast the two, and thus check whether or not the theory matches the facts.

The preceding argument carries over to the scatters or mean standard deviations occurring in the Heisenberg inequality. Instead of saying that they are the effects of measurements, the realist will say that they are objective spreads around the respective averages. All of which presupposes the realist interpretation of probability as a measure of chance, not of our uncertainty or ignorance. One may be very certain about a variance, and very uncertain about a sharp value. But physics, whether quantum or classical, is indifferent to our states of mind – a subject for psychology.

The rationale for the realist interpretation of the quantum theory is the following. First, quantum mechanics is not a mechanics proper, since it does not include the concept of point-like position nor, consequently, that of orbit – except of course as averages. Hence it is naive to expect that the similarities between quantum mechanics and classical mechanics are other than the formal analogies between, for instance, the corresponding energy operators (or hamiltonians).

Second, the general axioms of the quantum theory do not contain any variables referring to experiments, let alone observers. Hence, it is fallacious to interpret the theorems, such as Heisenberg's inequality, in terms of measurements. True, the von Neumann postulate does refer to measurements. But, as argued earlier, it is not plausible because it assumes the existence of a universal meter. Besides, it begs the question about the collapse of the state function, a process we would like to understand instead of having to accept as a brute fact. For these reasons it is not advisable to keep it as an axiom.

Third, the standard deviations occurring in Heisenberg's inequality and its kin subsist even in the case of the free quanton and, in particular, for a quanton on which no measurement is being performed. Hence, they must correspond to an objective spread of the quanton – which of course the experimenter can either shrink or stretch by the simple expedient of compressing or expanding the container.

10. Conclusion

To sum up, quantum physics is twenty-five centuries old, not just one. Moreover, and this is crucial, the trademark of the new physics is not quantization, since this is also a property of things as ordinary as drums, elastic beams, electrically charged clouds, and batteries.

What is peculiar to quantum physics is that it describes accurately things that are alien to everyday experience, namely objects lacking in precise positions, shapes, velocities, and energies. It also describes strange things such as a vacuum with physical properties, and couples that stay together even after divorce. All these things are certainly extraordinary, but none of them is spooky and beyond the experimenter's reach. They are just counter-intuitive.

However, the main trouble with the quantum theory is not that it is counterintuitive: all radically new theories share this feature. The main trouble with that theory is that it was yoked from the start to a philosophy incongruent with it because it is centered in the inquirer and her perceptions rather than in the real world. But such contamination of the new physics by an old philosophy was perhaps unavoidable at the time when the quantum theory was being built. Indeed, that philosophy, logical positivism, was less backward than its most popular rivals: intuitionism, neo-Kantianism, neo-Hegelianism, neo-Thomism, phenomenology, existentalism, and even dialectical materialism.

Logical positivism was slightly more advanced than its rivals because it called itself scientific (even though it was not), it stressed the need for empirical testability, it demanded conceptual precision, and in particular it embraced modern logic, rejected by all its rivals. No wonder then that logical positivism, or neopositivism, was adopted at that time by nearly all physicists, in particular the fathers of the quantum theory – even by Niels Bohr, who had initially been attracted to the nebulous and pessimistic musings of his countryman, Søren Kierkegaard.

This explains the emergence of the vicious circle:

Philosophy at time $t - 1 \rightarrow$ *Science at time* $t \rightarrow$ *Philosophy at time* $t - 1$,

instead of the virtuous helix

Philosophy at time $t \rightarrow$ *Science at time* $t \rightarrow$ *Philosophy at time* $t + 1$.

At the time or the birth of quantum mechanics, the physicists hardly noticed the main backward features of neopositivism. This was its retention of the subjectivism and phenomenalism characteristic of Berkeley, Hume, and Mach. This ingredient was decisive in the formulation of the Copenhagen interpretation, particularly its thesis that every microphysical event is the product of some measurement, so that every probability one calculates must be the probability of finding something upon performing a measurement.

This philosophy was criticized by Einstein, de Broglie, Schrödinger, Planck, and Bohm. All five favored not only realism but also causalism and classicism. Regrettably, they mixed up these three features, although they are clearly distinct. Consequently, the experimental refutation of Bell's inequalities was misinterpreted as a joint refutation of the three theses in question, while in fact realism was not affected.

Indeed, an analysis of any physical experiment shows that the experimenter assumes the independent existence of the thing he intends to observe, measure, or alter, as well as that of the tools he uses. Were it not so, the confrontation of theoretical predictions with experimental data would make no sense. In particular, the experimenter could not claim that he had made any discoveries: he would have to say instead that he invented or constructed everything – which would sound either schizophrenic or postmodern.

To conclude. Quantum physics has turned twenty-five centuries old, and it continues to pose intriguing philosophical problems. And, far from having toppled realism and determinism, it has enriched them. It has done so by showing that the world is far more complex and strange than it appears to be; that reality cannot be adequately described in ordinary language, just as melodies cannot be translated into words; that causality intertwines with chance; and that the discovery of new facts goes hand in hand with the invention of new ideas and new experimental techniques.

Moral 1: The scientists who do not update their philosophy contaminate their science with cadaverous philosophies. *Moral 2*: The philosophers and sociologists of science who do not update their science are doomed to talk to the dead and confuse the living.

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